2101001102020001
EXAMINATION FEBRUARY-MARCH 2024
MASTER OF ARTS (MATHEMATICS) (PART - II) (EXTERNAL)
NUMERICAL ANALYSIS – (502)-LEVEL 2

[Time: As Per Schedule] **[Max. Marks: 100]**

Instructions:

1. Fill up strictly the following details on your answer book

a. Name of the Examination : **MASTER OF ARTS (MATHEMATICS) (PART - II) (EXTERNAL)**

b. Name of the Subject : **NUMERICAL ANALYSIS - (502) - LEVEL 2**

c. Subject Code No : **2101001102020001**

2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. There are five questions in this question paper

Seat No:

Student's Signature

Q.1

A. Define root and multiple root. Consider first degree equation $f(x) = a_0 x + a_1; a_0 \neq 0$. Then discuss Bisection method to solve the equation. **7**

B. Find the root of the equation $2x = cosx + 3$ using Aitken's Δ^2 method. Compute two steps **7**

C. Define rate of convergence and show that Newton-Raphson method has second order convergence. **6**

OR

A. Define shift and average operator and prove the following result **7**

a) $\mu = \frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right)$

b) $\Delta\nabla = \Delta - \nabla$.

B. Derive general Iteration method and also necessary condition for convergence of the method. **7**

C. Define rate of convergence and show that rate of convergence for Secant method is $p = 1.618$. **6**

Q.2 A. Define Central difference operator and prove the following results **7**

a) $\delta = \left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right)$

b) $f(x+nh) = f(x) + \binom{n}{1}\Delta f(x) + \binom{n}{2}\Delta^2 f(x) + \cdots \ldots + \binom{n}{n}\Delta^n f(x)$

B. Construct Hermite interpolation polynomial that fits data and Estimate value of $f\ (0.75)$. **7**

x	0	0.5	1.0
$f(x)$	0	0.4794	0.8415
$f'(x)$	1.0000	0.8776	0.5403

C. The following data represents the function $f(x) = cos(x + 1)$. **6**

x	0.0	0.2	0.4	0.6
$f(x)$	0.5403	0.3624	0.1700	-0.0292

Estimate the value of $f\ (0.5)$ using Newton's backward difference interpolation.

OR

A. Find the solution of the given system using Cholesky decomposition method. $4x_1 - x_2 - x_3 = 3$; $-x_1 + 4x_2 - 3x_3 = -0.5$; $-x_1 - 3x_2 + 5x_3 = 0$ **7**

B. Find Jacobian matrix for the following system of equations **7**

$f_1(x,y) = x^2 - y^2 + xy = 0, f_2(x,y) = 3x^2 + 5y^2 + x = 0\ \ at\ (1,2)\ \&\ (0.5,1)$

C. Find the inverse of the coefficient matrix of the system by Gauss-Jordan method with partial pivoting. $\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 2 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$ **6**

Q.3 A. Perform three iterations of the Newton-Raphson method to solve the system of non-linear equations $x^2 + xy + y^2 = 7$; $x^3 + y^3 = 9$ Take the initial approximation $(x_0, y_0) = (1.5, 0.5)$. **7**

B. Find the solution of the given system using Cholesky decomposition method. **7**

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

C. Derive explicit multistep method **6**

OR

A. Derive the formula for the first derivative $y = f(x) = sin\, x\ of\ O(h^2)$ using Newtons Backward difference approximation. Estimate $f'(\pi/4)$ with $h = \pi/2$. Obtain the bound on the truncation error and compare with the exact solution. **7**

B. The following data for $f(x) = x^4$ is given **7**

X	0.4	0.6	0.8
F(X)	0.0256	0.1296	0.4096

Find $f'(0.8)$ and f'' (0.8) using quadratic interpolation

C. Derive the extrapolation formula for the numerical differentiation rule $f'(x_0) \approx \frac{1}{2h}\,[f(x_0 + h) - f(x_0 - h)]$ **6**

Q.4 A. Find the approximate value of $\int_0^1 \frac{sin\, x}{x} dx$ using mid-point and three-point rule **7**

B. Find the largest eigenvalue in modulus and the corresponding eigenvector of the matrix $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ using the Power method **7**

C. Find all eigenvalues and the corresponding eigenvector for the matrix: **6**

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

OR

A. Evaluate $I = \int_0^1 \frac{x}{x^3+10} dx$ using Trapezoidal and Simpson rule with number of points taken as 3. **7**

B. Evaluate $I = \int_0^\infty (3x^3 - 5x + 1)\, e^{-x} dx$ using gauss - Laguerre two point formula **7**

C. Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ using simpson's rule using 3, 5 and 8 nodal points and obtain a bound for the error. The exact value of $I = \ln 2 = 0.693147$ correct up to six decimal places. **6**

Q.5 A. For the initial value problem $u' = t^2 + u^2, u(0) = 0$ determine the first three non-zero terms in the Taylor series for u(t) and find u(1). Also determine t when the error in u(t) obtained from the first two non-zero terms is to be less than 10^{-6} after rounding. **7**

B. For the initial value problem $u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$. Find $u(0.4)$ using Heun method. **7**

C. Solve the boundary value problem $u" = u + x, u(0) = 0, u(1) = 0$ with $h = 1/4$. Using the second order finite difference method. **6**

OR

A. Solve the boundary value problem $y''(x) = -y(x), y(0) = 0, y(1) = 1.1752$ using shooting method. **7**

B. Solve the boundary value problem $(1 + x^2)y'' + \frac{4x}{1+x^2} y' + \frac{2}{1+x^2} y = 0$ with the boundary condition $y(0) = 1$ and $y(2) = 0.2$. Use the finite difference scheme to determine the value of $y(1)$. Compare your answer with the exact value obtain from the analytic solution $y = \frac{1}{(1+x^2)}$ **7**

C. Derive the first order forward and backward difference scheme. Also find the truncation error of the scheme **6**

*****